Comments on "Strength and fracture properties of asbestos–cement mortar composites"

In a previous communication in this journal [1] fracture mechanical properties of asbestos–cement mortar composites were presented. In this paper it was concluded that the critical stress intensity factor, K_c, determined on a notched three-point bend specimen, may be regarded as a usable material property.

For asbestos–cement mortar composites it is shown [1] that the fibres are extracted from the cement paste matrix at crack propagation. When a notched specimen of this material type is subjected to load, one may assume a stress-distribution according to Dugdale [2] in front of the notch, see Fig. 1.

In [3], fracture mechanical calculations are carried out showing the dependence of specimen depth on the measured value of K_c. This dependence, for a stress-distribution according to Dugdale, is presented in Fig. 2 where the relationship between $K_c'/\sqrt{G_cE_t}$ and the dimensionless parameter $W/(G_cE_t/f_t^2)$ is shown, where W is the beam depth, G_c is the fracture energy, E_t is the tensile Young's modulus, f_t is the tensile strength and K_c' is the value of K_c estimated from the maximum load at a three-point bend test on a notched specimen. G_c is independent of specimen size and is equal to the specific fracture energy, R, [1], where R is determined in a stable three-point bend test, i.e. in this case when $a/W > 0.5$ (where a is the notch depth).

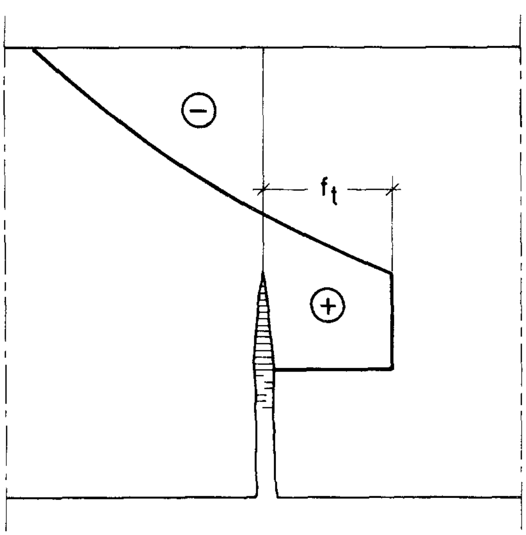

Figure 1 A stress-distribution according to Dugdale in front of the notch.

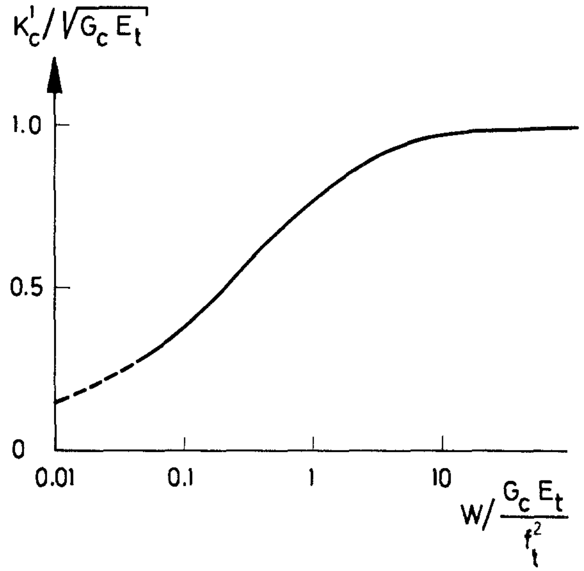

Figure 2 $K_c'/\sqrt{G_cE_t}$ as a function of $W/(G_cE_t/f_t^2)$ for a stress-distribution according to Dugdale.

In [1] the parameters W, G_c, E_t and f_t are determined for different mass fractions of fibres, see Table I.

The values in Table I make it possible to estimate values of K_c from $\sqrt{G_cE_t}$ and Fig. 2. In Table II these values are compared with the values of K_c' that were determined in laboratory tests [1].

As seen in Table II, the theoretical values of K_c' and those determined experimentally in laboratory tests [1] are in agreement. This implies that the curve in Fig. 2 is relevant for asbestos–cement mortar composites and that K_c' for normal dimensions is too dependent on specimen size to be useful as a material property. The results also

TABLE I Material parameters for different asbestos–cement mortar composites

Mass fraction of fibres	W (mm)	f_t (MPa)	E_t (MPa)	G_c ($N\,m^{-1}$)
0.05	26	7.1	17200	760
0.10	26	10.8	17200	2300
0.15–0.20	26	15.8	17200	3300

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TABLE II K_c' values determined from $\sqrt{G_c E_t}$ and Fig. 2, and from tests respectively

Mass fraction of fibres	$W/(G_c E_t/f_t^2)$	$\sqrt{G_c E_t}$ ($MN\,m^{-3/2}$)	K_c' (from $\sqrt{G_c E_t}$ and Fig. 2) ($MN\,m^{-3/2}$)	K_c' test results [1] ($MN\,m^{-3/2}$)
0.5	0.10	3.6	1.4	1.5
0.10	0.08	6.3	2.2	2.2
0.15–0.20	0.11	7.5	3.0	3.2

imply that linear elastic fracture mechanics are unsuitable for this material. Other calculation methods have to be used and in [4] a model, the Fictitious Crack Model is presented which is probably very suitable for fracture mechanical calculations, where fibre composites are concerned.

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Reply to 'Comments on "Strength and fracture properties of asbestos–cement mortar composites" '

I would like to thank Dr Petersson for his useful comments [1] on my paper referred to above [2]. The point concerning the size dependence of the critical stress intensity factor (K_c) is a valid one. The problem was realized soon after the paper [2] was published. In [2], K_c was defined as that K evaluated at crack initiation load which for the specimen size chosen was fortuitously close to the maximum load, this is, where K_c equals K_i. In larger specimens it was shown that K_c at maximum load was larger than K_i. The size effects on the fracture behaviour of asbestos–cements have been recently studied by the present author and the results are published elsewhere [3]. It was concluded in [3] that a single fracture parameter such as K_c was inadequate to describe the total fracture behaviour of asbestos–cement. Instead, a K_R-curve approach was suggested for the analysis [3, 4]. This was because as the crack grew increasing amounts of fibres were pulled out, bridging the crack opening behind its tip, giving a rising crack growth resistance (K_R) curve until fibre reinforcement reached a steady state when the K_R-curve reached a plateau. Whether linear elastic fracture mechanics can be used with the K_R-curve technique depends largely on whether the damage zone, r, over which the fibres are bridging is small compared to the notch depth, a. If the ratio r/a is not small an elasto-plastic J_R-curve analysis has to be used. From the experimental results of [3, 4] it seems, however, that LEFM is applicable and that the K_R-curve is a material property, independent of specimen size and initial notch depth (see Figs. 7 to 9 of [3]). It should be pointed out that while K_i does not depend on specimen size the largest attainable K does. The plateau K_R cannot be obtained unless the size is big enough to allow the full K_R-curve to be measured. For the asbestos–cement studied in [3], using three-point notched beams, the beam depth, W, must be at least 200 mm for the ratio a/W to equal 0.30. Further work on the geometry dependence of the K_R-curve is in progress.

With regard to Dr Petersson's comments [1], firstly, I must apologise for the confusion in the use of symbols because, as explained above, K_c

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defined in [2] is really K_i evaluated at crack initiation of the reinforced matrix. For asbestos–cements K_i is a material property but K_c evaluated at *maximum load* is not because its magnitude increases with specimen size as shown in [3] and noted by Dr Petersson. Moreover, the maximum load K_c does not correspond to the plateau K_R of the K_R-curve. Secondly, it is stated in [2, 3] that $K_c^2 = EG_c$ only when both K_c and G_c refer to *crack initiation*. If G_c is measured from the total work under the load–deflection diagram of a stable three-point bend test on a notched beam, it represents only an *average* specific fracture energy comprising both crack initiation and crack propagation. K_c, calculated from $\sqrt{(EG_c)}$, thus represents only an *average* stress intensity factor. Such a parameter is less useful than a K_R-curve which is able to account for the slow crack growth phenomenon observed even in notched beams with $W = 400$ mm. Thirdly, it seems that E_b instead of E_t should be used in Table II in Dr Petersson's discussion [1] because the three-point notched beams are subjected to bending. Because $E_b = \frac{1}{2}E_t$ the predicted K_c' values from Dr Petersson's analysis and those obtained in [2] will not show the same kind of good agreement as given in his Table II.

In summary, I fully agree with Dr Petersson that the maximum load K_c is too dependent on specimen size to be a useful material property. Unless G_c for crack initiation and crack propagation are identical, which for asbestos–cements they are not, I am not convinced that the true K_c can be simply obtained from $\sqrt{(EG_c)}$, where G_c is obtained from the work of fracture method. To characterize the complete fracture behaviour of asbestos–cements, from initiation, to propagation and to eventual failure I believe that the K_R-curve approach is the most suitable and useful method. We are also currently investigating the G_R-curve approach by considering incremental work dissipation in the fibre pull-out region as the crack slowly extends. In this respect Dr Petersson's Fictitious Crack Model may be useful [5, 6].

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On the validity of the Dugdale model for craze zones at crack tips in PMMA

In contrast to previous results [1, 2] of interference optical measurements of the craze zone at crack tips in PMMA loaded under Mode-I-conditions, Israel *et al.* [3] report in their recent paper that the Dugdale model is not fully adequate to describe craze geometries in PMMA and from this they suggest a modified craze zone model. They base this hypothesis on their finding that the plastic zone, as calculated from the Dugdale model using constant values of Young's modulus and yield stress and their stress intensity factors for the DCB specimen, is larger (by a factor of about 2.5) than the interference optically measured craze zone. The profile of the craze zone and the Dugdale plastic zone, however, are found to be very similar.

The purpose of this communication is to show that: the Dugdale model describes the profile and size of craze zones in PMMA quite well and gives information about the viscoelastic material behaviour; to examine the discrepancy reported by Israel *et al.*; and to point out some facts suggesting that the authors erred in their determination of K_I.

There is agreement with the authors that in such investigations it is very important to measure

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